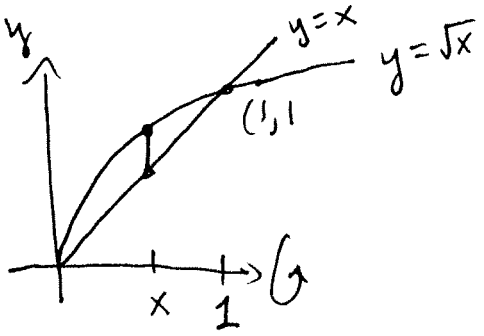
Math 172A
Spring 2010
Instructor: Shawn Rafalski

Integral Calculus
Quiz 2
Write your name on this quiz

Solution

Let D be the region bounded by the curve $y = \sqrt{x}$ and the line $y = x$. For each part below, write down an integral (including the limits of integration) that represents the volume of the solid of revolution obtained by revolving D around the given line. Then **choose one** of these integrals and evaluate it.

1. The x–axis.

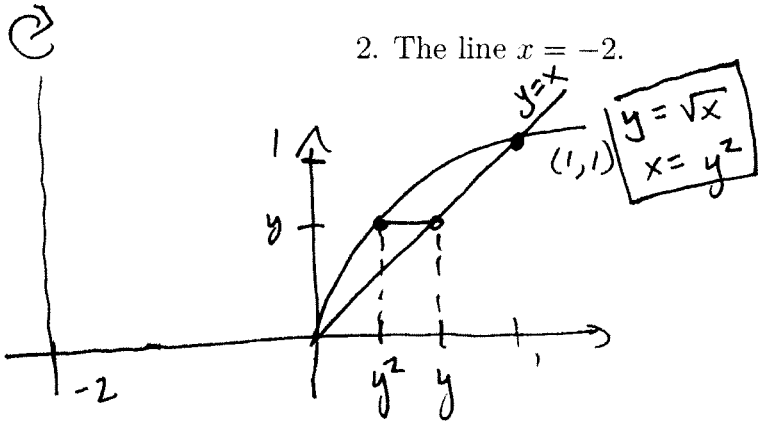


$$\int_0^1 \pi\left[(\sqrt{x})^2 - x^2\right] dx$$

$$= \pi\int_0^1 (x - x^2)\,dx = \pi\left[\frac{x^2}{2} - \frac{x^3}{3}\right]\Big|_0^1$$

$$= \pi\left(\frac{1}{2} - \frac{1}{3}\right) = \boxed{\pi/6}$$

2. The line $x = -2$.



$$\int_0^1 \pi\left[\left[y-(-2)\right]^2 - \left[y^2-(-2)\right]^2\right] dy$$

$$= \pi\int_0^1 \left[(y+2)^2 - (y^2+2)^2\right] dy$$

$$\pi\int_0^1 (4y - 3y^2 - y^4)\,dy$$

$$= \pi\left[2y^2 - y^3 - \frac{y^5}{5}\right]_0^1 = \pi\left[1 - \frac{1}{5}\right] = \boxed{\frac{4\pi}{5}}$$