Quiz 5 | Math 172 | Spring 2010

(1) $\lim_{x\to\infty} \left(\frac{\ln x}{x}\right)^2 \qquad \left(\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty} \frac{2\ln x \cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{2\ln x}{x} \qquad \left(\text{still } \frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty} \frac{2/x}{1} = \lim_{x\to\infty} +2/x = \boxed{0}$

(2) $\lim_{x\to 0} \frac{e^x - 1}{x} \qquad \left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{e^x}{1} = e^0 = \boxed{1}.$

(3) $\lim_{x\to 0} \frac{\sin x}{x^2+1} = \frac{\sin 0}{0^2+1} = \frac{0}{1} = \boxed{0}.$

(4) $\lim_{x\to 0^+} x^x \qquad (0^0)$

$$y = x^x \iff \ln y = x \ln x$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x}$$

$\left(\frac{-\infty}{\infty}\right)$

$$= \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = 0$$

So $\lim_{x\to 0^+} \ln y = 0$, therefore $y = x^x \to e^0$

$$\| \quad 1$$

So $\lim_{x\to 0^+} x^x = \boxed{1}$.

(5) $$\int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx$$

$u = x^2 \quad v = e^x$

$du = 2x\,dx \quad dv = e^x dx$

$u = 2x \quad v = e^x$

$du = 2\,dx \quad dv = e^x dx$

$$= x^2 e^x - \left[2x e^x - \int 2 e^x \, dx\right]$$

$$= \boxed{x^2 e^x - 2x e^x + 2e^x + C}$$