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Math 272A
Spring 2010
Instructor: Shawn Rafalski

Multivariable Calculus II
Exam 1
Solutions

Write your name on this exam right now. Your work on this exam is to be your work alone. No calculators allowed. You have one hour to finish. Explain your answers clearly, and *show your work.* This exam has 9 pages, and the questions are worth a total of 100 points (not including bonus points). Only work on the bonus questions **after** you have tried to do all the regular questions. Don't forget to breathe regularly, and good luck!!

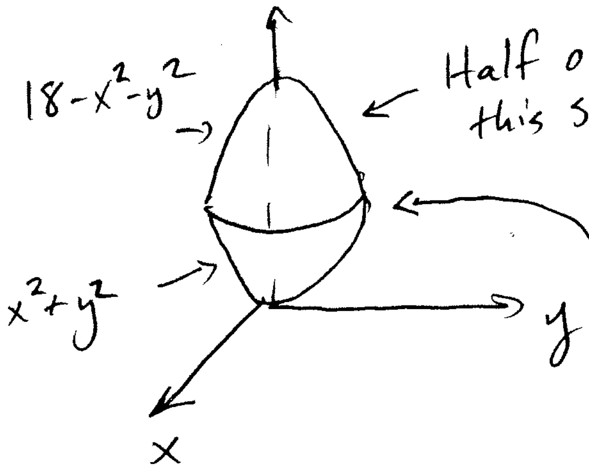
Here is some info you may (or may not) need for this exam.

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$
$$dV = \rho^2 \sin\phi \, d\rho d\phi d\theta$$
$$\cos^2 x = \frac{1+\cos(2x)}{2} \qquad \sin^2 x = \frac{1-\cos(2x)}{2}$$

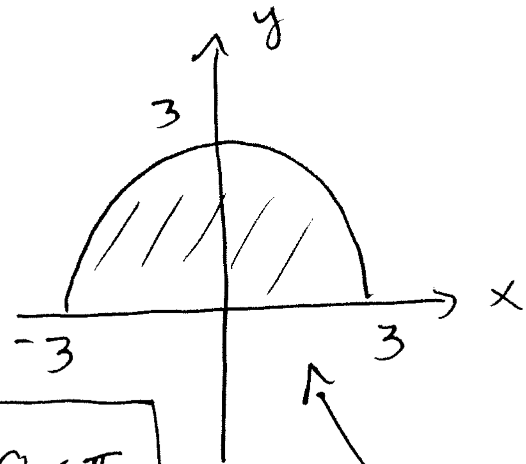
Begin working on the next page.

Cylindrical

1. (20 points) Compute the volume of the part of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$ where the y-coordinate is *non-negative.*



Half of this solid



$$\boxed{\begin{array}{l} 0 \le \theta \le \pi \\ 0 \le r \le 3 \end{array}}$$

intersection: $\underbrace{18 - x^2 - y^2 = z = x^2 + y^2}$

$r^2 = x^2 + y^2 = 9$

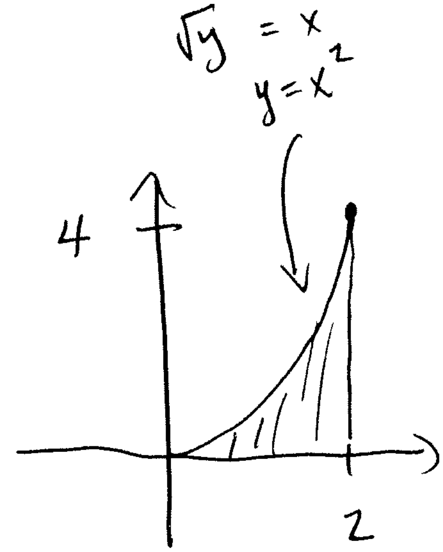
Volume

$$= \int_0^{\pi}\int_0^{3}\left[\int_{r^2}^{18-r^2} dz\right] r\,dr\,d\theta = \cdots = \boxed{\frac{81\pi}{2}}$$

2. (12 points) Reverse the order of integration of the integral

$$\int_0^2 \int_0^{x^2} f(x,y)\, dy dx.$$

$$\left.\begin{aligned} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{aligned}\right\}$$

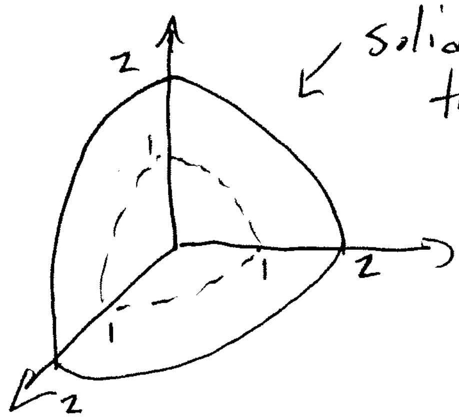


$$\left\{\begin{aligned} 0 \le y \le 4 \\ \sqrt{y} \le x \le 2 \end{aligned}\right.$$

$$\boxed{\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\, dx\, dy}$$

Spherical

3. (20 points) Let E be the solid bounded by the spheres of radii 1 and 2 in the first octant. Suppose the density function on E is $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, and suppose that the mass of E with this density function is equal to $15\pi/8$. Write down ***but do not evaluate*** a triple integral (including the limits of integration and the appropriate choice of coordinates) that represents the z-coordinate of the center of mass of E. (**Bonus 2 points:** Evaluate this integral).



solid between these :

$0 \le \theta \le \pi/2$

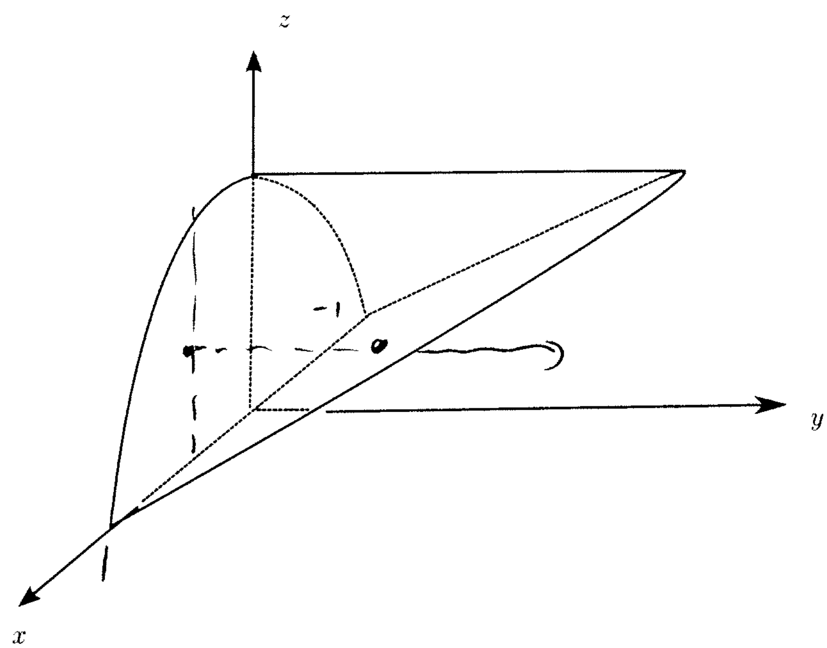
$0 \le \phi \le \pi/2$

$1 \le \rho \le 2$

$$\bar{z} = \frac{1}{\text{mass}} \iiint_E z\, \delta(x,y,z)\, dV$$

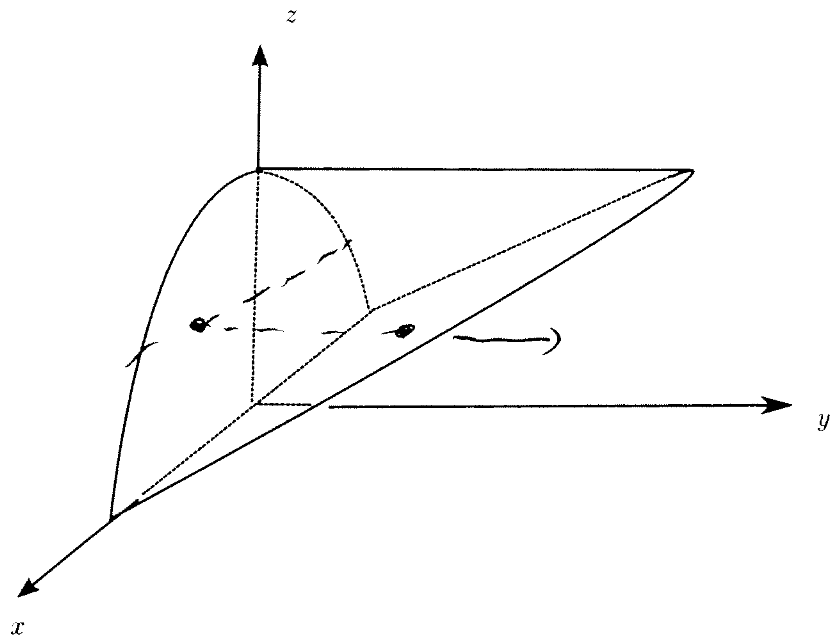
$$= \boxed{\frac{8}{15\pi} \int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho\cos\phi \cdot \sqrt{\rho^2} \cdot \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta}$$

$$= \cdots = \boxed{62/75}$$



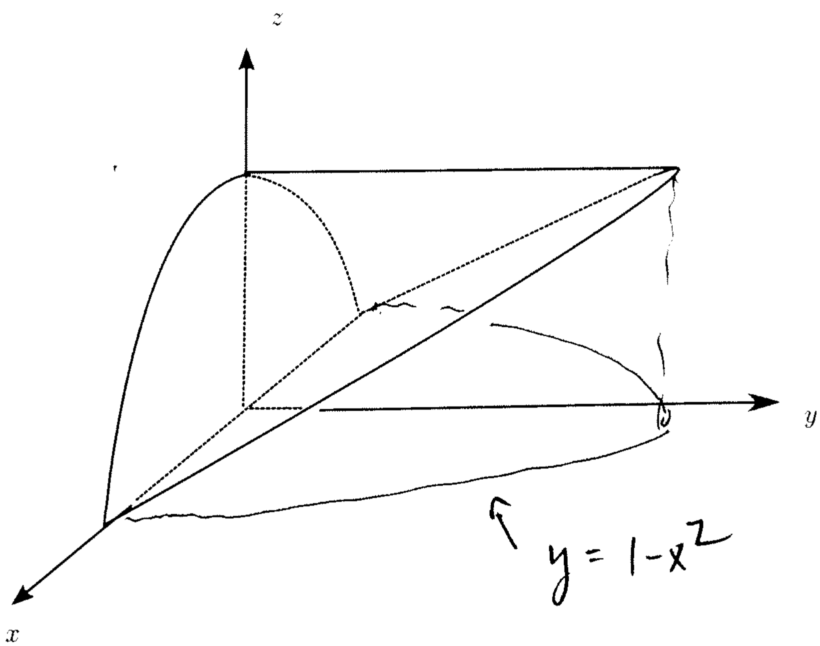
4. (12 points each) The figure above shows the solid bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $y = z$ and $y = 0$. For each part below, fill in the limits of integration so that the integral represents the volume of this solid. (For your convenience, the figure is reproduced at the top of each part of this exercise).

(a)
$$\int_{-1}^{1} \int_{0}^{1-x^2} \int_{0}^{z} dydzdx$$



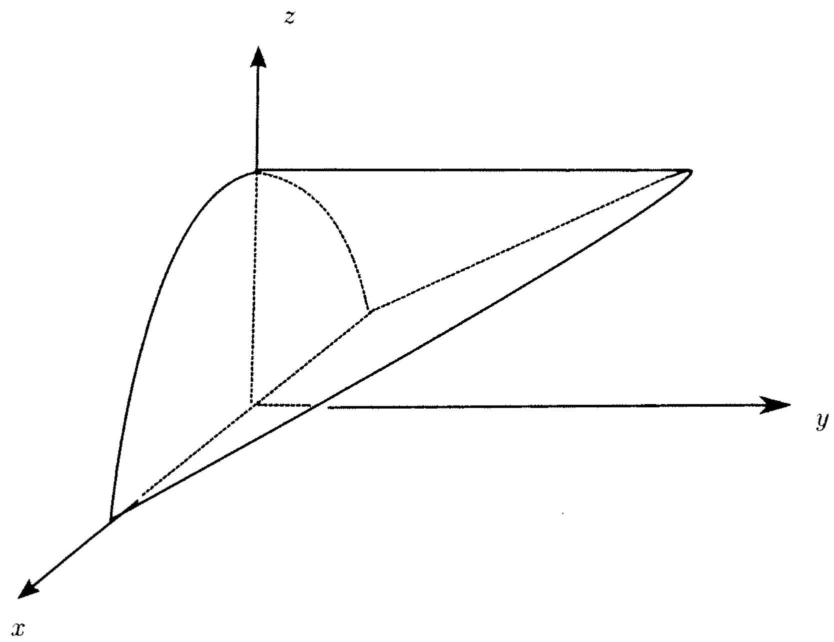
(b)

$$\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_0^z dy\,dx\,dz$$



(c)

$$\int_{-1}^{1}\int_{0}^{1-x^2}\int_{y}^{1-x^2} dzdydx$$



(d)

$$\int_0^1 \int_0^z \int_{-\sqrt{1-z}}^{\sqrt{1-z}} dx\,dy\,dz$$

5. (**Bonus 1 point each**) What is the name of Google's search algorithm? In the participation exercise at the end of the Google talk, where inside of the 10×10 array of numbers did the counting exercise end for both the student and Prof. D?

(1) PageRank

(2)