# Worst. Exam. Ever. j/k...

Math 272A
Spring 2010
Instructor: Shawn Rafalski

Multivariable Calculus II
Exam 2

**Write your name on this exam right now.** Your work on this exam is to be your work alone. No calculators allowed. You have one hour to finish. Explain your answers clearly, and *show your work*. This exam has 9 pages, and the questions are worth a total of 100 points (not including bonus points). Only work on the bonus questions **after** you have tried to do all the regular questions. Don't forget to breathe regularly, and good luck!!

Solutions

Begin working on the next page.

1. (12 points each) Set up each of the line integrals below, including the limits of integration. You do not need to evaluate these integrals (yet).

   (a) $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $F(x, y, z) = z\mathbf{i} + y\mathbf{j} - x\mathbf{k}$ and $\mathbf{r}(t) = t\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$ for $0 \le t \le \pi$.

$$\int_0^{\pi} \langle \cos t, \sin t, -t \rangle \cdot \langle 1, \cos t, -\sin t \rangle \, dt$$

$$= \int_0^{\pi} (\cos t + \sin t \cos t + t \sin t) \, dt = \boxed{\pi}$$

   (b) $\int_C (x + yz)\, dx + 2x\, dy + xyz\, dz$, where $C$ consists of the line segment from $(1, 0, 1,)$ to $(2, 3, 1)$ and the line segment from $(2, 3, 1)$ to $(2, 5, 2)$.

$C = C_1 \cup C_2$

$C_1$: $\vec{r}(t) = \langle 1 + t, 3t, 1 \rangle$

$C_2$: $\vec{r}(t) = \langle 2, 3 + 2t, 1 + t \rangle$

$0 \le t \le 1$

$$\int_C = \int_{C_1} + \int_{C_2} = \int_0^1 (7 + 10t)\, dt + \int_0^1 (4t^2 + 10t + 14)\, dt$$

$$= \boxed{\frac{97}{3}}$$

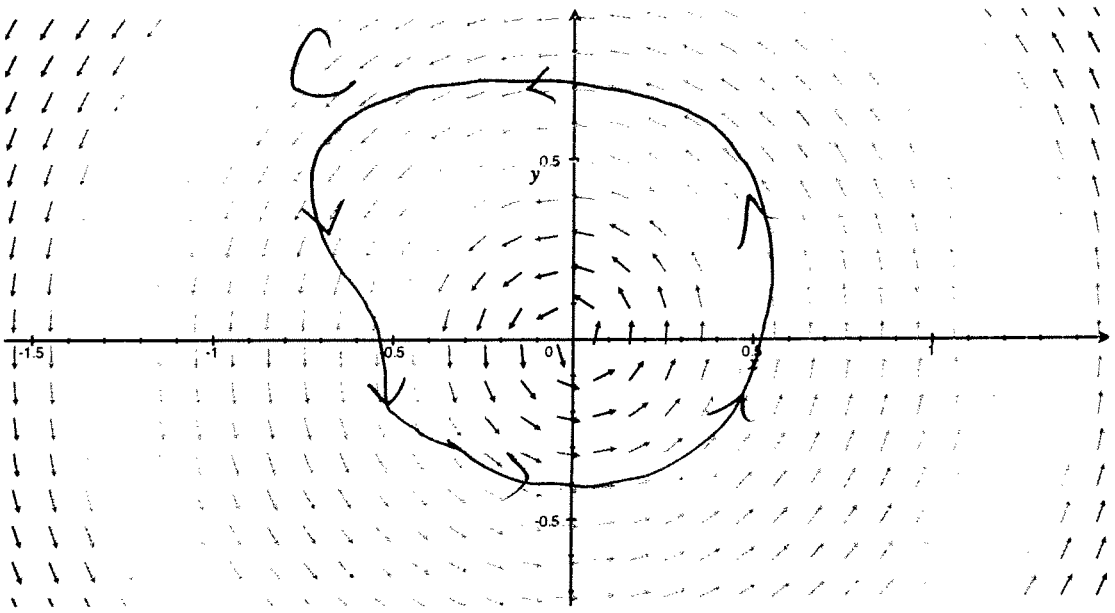
(c) $\int_C xy^4\,ds$, where $C$ is the right half of the circle $x^2 + y^2 = 16$.

$C: \vec{r}(t) = \langle 4\cos t, 4\sin t \rangle \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$$\int_{-\pi/2}^{\pi/2} \left(4^5 \cdot \cos t \cdot \sin^4 t \cdot 4\sqrt{\sin^2 t + \cos^2 t}\right) dt$$

$$= \boxed{\frac{2}{5} \cdot 4^6}$$

2. (12 points) Evaluate one of the three integrals from the previous exercise.




3. (5 points) Is the vector field pictured above a conservative vector field? Explain your answer.

No. The path shown has

$$\oint_C \vec{F} \cdot d\vec{r} > 0$$

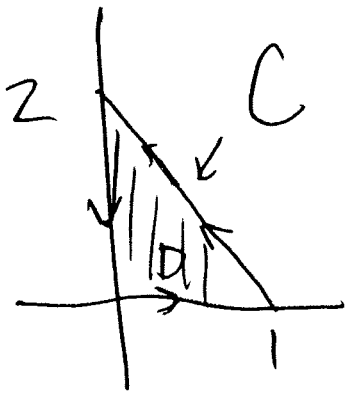
and it would be $= 0$ if $\vec{F}$ was conservative.

4. (15 points) Evaluate the line integral

$$\oint_C (2y - e^x \sin(x^2))\,dx + (\cos(y^{47}) + 5x)\,dy,$$

where $C$ is the boundary of the triangle with vertices $(0,0), (1,0)$ and $(0,2)$ in the plane, oriented counterclockwise.

$Q_x - P_y = 5 - 2 = 3$




So

Green's Thm

$$\oint_C \;\overset{\downarrow}{=}\; \iint_D 3\,dA = \boxed{3}$$

5. Assume that the vector field $\mathbf{F}(x, y, z) = \langle y + 2ze^x, x - 2y, 2e^x \rangle$ is conservative.

(a) (12 points) Find a potential function $f(x, y, z)$ for $\mathbf{F}$.

$f_x = y + 2ze^x$

$f_y = x - 2y$

$f_z = 2e^x$

$f_z = 2e^x \rightarrow f = 2ze^x + g(x,y)$

$2ze^x + \frac{\partial g}{\partial x} = f_x = y + 2ze^x$

$\frac{\partial g}{\partial x} = y \leftarrow g(x,y) = xy + h(y)$

$f = 2ze^x + xy + h(y)$

$x - 2y = f_y = x + h'(y) \rightarrow h'(y) = -2y$

$h(y) = -y^2 + C$

$f(x,y,z) = 2ze^x + xy - y^2 + C$

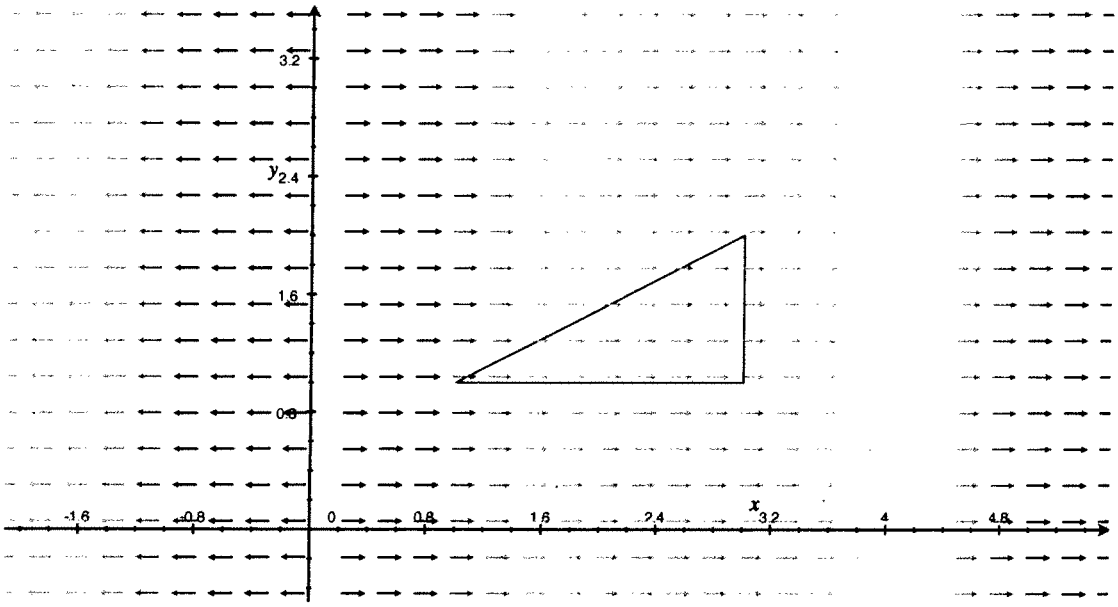
(b) (8 points) Use your answer to part (a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the straight line segment from $A(2, -1, 0)$ to $B(0, 1, 1)$.

$$\int_C \vec{F} \cdot d\vec{r} = f(0,1,1) - f(2,-1,0)$$

Fund. Thm of Line Int.'s

$$= 1 - (-3)$$

$$= 4$$




6. The figure above depicts a vector field $\mathbf{F}(x, y)$ and the closed (triangular) path $C = C_1 \cup C_2 \cup C_3$, where $C_1, C_2$ and $C_3$ are the horizontal, vertical, and diagonal line segments, respectively. Suppose $C$ is oriented *counterclockwise.*

   (a) (6 points) Is the work done by $\mathbf{F}$ along the horizontal segment $C_1$ positive, negative or zero? Explain your answer.

Positive. $\vec{F}$ acts in the direction of the curve.

   (b) (6 points) Suppose $\mathbf{F}$ is a conservative vector field. Is the work done by $\mathbf{F}$ along the path $C_3 \cup C_1$ (that is, the path down the hypotenuse of the triangle, and then horizontally) positive, negative or zero? Explain your answer.

$\vec{F}$ cons. $\Rightarrow \oint_C \vec{F} \cdot d\vec{r} = 0$ b/c $C_2 \perp \vec{F}$

but $\int_{C_3 \cup C_1} \vec{F} \cdot d\vec{r} = -\int_{C_2} \vec{F} \cdot d\vec{r} = 0$ so

7. (**Bonus 1 point each**) You can only earn bonus points if you attended the Chess colloquium talk by Dr. King. So check here _____ if you were able to attend.

   (a) Name one of the three brands of clothing that Dr. King thinks the typical college student wears.

   (b) What was the name of the computer in the Human vs. Computer chess match discussed in Dr. King's talk?