Math 272 | Spring 2010 | Quiz 4

Set up the following three integrals, and evaluate one of the first two.

(1) $\int_C xyz^2\,ds$, where C is the line segment from $P(-1,5,0)$ to $Q(1,6,4)$.

Soln:

$C:\ \vec{r}(t) = \vec{OP} + t\vec{PQ} = \langle -1+2t,\ 5+t,\ 4t\rangle$

$(0 \le t \le 1).\quad \int_0^1 (-1+2t)(5+t)(4t)^2 \cdot \sqrt{(2)^2+(1)^2+(4)^2}\,dt$

$= 16\sqrt{21}\int_0^1 \left(2t^4 + 9t^3 - 5t^2\right)dt$

$= 16\sqrt{21}\cdot\left[\frac{2}{5}t^5 + \frac{9}{4}t^4 - \frac{5}{3}t^3\right]\Big|_0^1 = \underbrace{16\sqrt{21}\left(\frac{2}{5}+\frac{9}{4}-\frac{5}{3}\right)}.$

don't have to simplify ☺

(2) $\int_C z\,dx + x\,dy + y\,dz$

where C: $\vec{r}(t) = \langle t^2, t^3, t^2 \rangle$ $\quad 0 \le t \le 1$

Soln:

$$\int_0^1 \left(t^2 \cdot 2t + t^2 \cdot 3t^2 + t^3 \cdot 2t \right) dt$$

$$= \int_0^1 \left(2t^3 + 5t^4 \right) dt = \left(\frac{t^4}{2} + t^5 \right) \Big|_0^1$$

$$= \boxed{3/2}$$

(3) $\int_C \vec{F} \cdot d\vec{r}$ where C: $\vec{r}(t) = \langle t^2, t^3, t \rangle$, $0 \le t \le 1$.

$\vec{F}(x,y,z) = \langle e^z, xz, x+y \rangle$.

Soln:

$$\int_0^1 \langle e^t, t^3, t^2 + t^3 \rangle \cdot \langle 2t, 3t^2, 1 \rangle \, dt$$

$$= \int_0^1 \left[2te^t + 3t^5 + (t^2 + t^3) \right] dt$$