Steve Sawin
Probability 3352

Problem Set III

Due March 5

All write-ups should be individually done Please tell me anyone you worked with.

1. Suppose your population is the integers from 1 to N, and you take a simple random sample of size n (without replacement) from it. Show that

   (a) [2pt] The mean of $\overline{X}$ is
   $$\mu_{\overline{X}} = \frac{N+1}{2}.$$

   (b) [3pt] The variance of $\overline{X}$ is
   $$\sigma^2_{\overline{X}} = \frac{(N+1)(N-n)}{12n}.$$

   Use the formulas in Appendix A

2. [2pt] Redo Question 5 from the previous Problem Set exactly, assuming it was sampling without replacement from a finite population of size 1000.

3. [4pt] Suppose that $X_1, X_2, \ldots, X_n$ are a random sample from an infinite population of a random variable with mean μ. Find the equation in the variables $a_1, \ldots, a_n$ that guarantees that the *statistic*

$$a_1 X_1 + a_2 X_2 + \cdots a_n X_n$$

is an unbiased estimator of μ.

4. [4pt] If a population has mean μ and variance σ^2 and if $X_1, \ldots, X_n$ is random sample from that population show that

$$\frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2$$

is an unbiased estimator of σ^2.

5. Suppose $X_1, X_2, \ldots, X_n$ are an independent sample of values from a population with mean μ_X and standard deviation σ_X. Let $\overline{X}$ represent the sample mean of the n samples. Similarly, let $Y_1, Y_2, \ldots, Y_m$ be independent samples from a population with mean μ_Y and standard deviation σ_Y. Let $\overline{Y}$ represent the sample mean of the m samples. Further assume that the Xs and Ys are independent.

   (a) [2pt] Derive $E[\overline{X} - \overline{Y}]$ giving justifications where needed.

   (b) [3pt] Derive $\text{Var}[\overline{X} - \overline{Y}]$ giving justifications where needed.

Note: This problem is based on problem 8.2 on page 272, but using different (less messy) notation.

Out of 20 points