

Problem Set V

Due April 9

All write-ups should be individually done, complete, neat and precise. Please tell me anyone you worked with or got help from and AI tools, websites or box you used and exactly how.

1. [1pt] Question 10.13, p. 326: Show if T is an unbiased estimator of θ and if $\text{Var}(T) \neq 0$ then T^2 is a *biased* estimator of θ^2 .

2. [3pt] Question 10.15, p. 326: Show the mean of a random sample of size n is a minimum variance unbiased estimator of the parameter λ for a Poisson distribution.

3. [3pt] Question 10.23, p. 327: Suppose that \bar{X}_1 and \bar{X}_2 are independent samples of size n_1 and n_2 respectively from the same normal population with mean μ and standard deviation σ . show that for each real number ω the following is an unbiased estimator of μ

$$\omega\bar{X}_1 + (1 - \omega)\bar{X}_2.$$

Show that *among these* the one with the minimum variance is the one with

$$\omega = \frac{n_1}{n_1 + n_2}.$$

4. [1pt] Question 10.51, p. 342: Given a random sample of size n from a Poisson distribution, use the method of moments to find an estimator for the parameter λ .

5. [2pt] Question 10.59, p. 342: Repeat the previous question using the method of maximum likelihood. Use the trick of minimizing the log of the likelihood.

6. [3pt] Suppose again that we are estimating μ from a normal population with σ known, using a simple random sample with sample mean \bar{x} . Suppose you want to pick your sample size n so that the margin of error (which is half the total width of the confidence interval) is equal to e . Give a formula for the minimum n that will accomplish this as a function of σ , e and z_α^* . Remember $z_\alpha^* = z_{(1-\alpha)/2}$ is the z -score such that α is the chance a standard normal random variable will fall between $-z_\alpha^*$ and z_α^* .

7. [2pt] Question 11.24, p. 370: A study of the annual growth of certain cacti showed that 64 of them, selected at random in a desert region, grew an average of 52.80 mm with a standard deviation of 4.5 mm. Construct a 99% confidence interval for the true average annual growth of the given kind of cactus.

8. [2pt] Question 11.38, p. 372: A sample survey at a supermarket showed that 204 of 300 shoppers regularly used cents-off coupons. Give a 95% confidence interval for the true proportion.

https://www.faculty.fairfield.edu/ssawin/stats/data/High_Jump.XLS

9. [3pt] Get the file at the link immediately above.

The first two columns give the heights jumped by a sample of men and another sample of women athletes. Use the two sample mean confidence interval to give a 90% confidence interval for the amount by which the average height jumped by men exceeds the average height jumped by women. Tell me what you would need to know about the population sizes, variables and sampling process to know that the assumptions for using this procedure were met.

Out of 20points.