

Test II

You may use two pages of notes, calculator and computer. Please sign your name below indicating that you have not offered or received help on this test to or from any person, nor used any resource other than your notes, me, and the Excel files on my website.

1. [8pt] Use the method of moments to find an estimator for σ^2 for a variable X .

2. [10pt] I sample 8 independent individuals from a population with a normal variable X , and compute \bar{X} and S . What is the probability that

$$\bar{X} - \mu > S/4?$$

Helpful Hint: For T a t -dist with $n - 1$ degrees of freedom, in Excel and TI-84 respectively $P(T > t) = \text{TDIST}(t, n - 1, 1) = \text{tcdf}(t, 99999, n - 1)$.

3. Let $f(x)$ be the pdf for the normal distribution with $\sigma = 1$, (I am setting $\sigma = 1$ to make the calculation as simple as possible, don't let it confuse you).

(a) [4pt] What is $f(x)$?

(b) [10pt] Find

$$\frac{1}{nE \left[\left(\frac{\partial \ln[f(x)]}{\partial \mu} \right)^2 \right]}?$$

(c) [7pt] Show \bar{X} is a *minimum variance, unbiased* estimator for μ if X is a normal variable with $\sigma = 1$.

4. You want to know how residents of the town of Fairfield feel about FU students, so you stop 60 people coming out of the Seagrape on Saturday night and ask them, and find that 48 of them like Fairfield students.

(a) [10pt] Find a 90% confidence interval for the proportion who like Fairfield students and express it in a sentence that explicitly includes the parameter, population and variable. (answer is a sentence with some numbers in it)

(b) [5pt] What does the 90% above mean? Specifically, interpret the confidence interval by identifying a set of things (probability space) and a statement about them (event) that is true for 90% of them. (answer is a short sentence)

(c) [6pt] Check each assumption of this procedure. (answer is a few words explaining why each is or is not met)

(d) [5pt] Identify a (pretty dramatic) source of sampling bias in the above study.

[12pt] A SRS of 50 Millennials asks how often in a week each used the “laughing-cry” emoji, and found $\bar{X} = 13.2$ and $S = 9$. A study asking 42 Gen Zers the same question found $\bar{X} = 8.9$ and $S = 7.1$. Find the 92% confidence interval for how much more Millennials use it than Gen-Z on average, stated in a complete English sentence.

5. A simple random sample of mathematics textbooks had the following number of pages:

580 1252 809 413 651 588 169 537 763 816.

(a) [5pt] Find the sample mean \bar{X} and the sample standard deviation S . To check you typed the info in correctly, I got $\bar{X} = 65?.80$, Where ? is a digit you still need to figure out.

(b) [12pt] Give the 96% confidence interval for the average number of pages of all math books in a complete English sentence.

(c) [6pt] Check all three assumptions.

out of 100 points