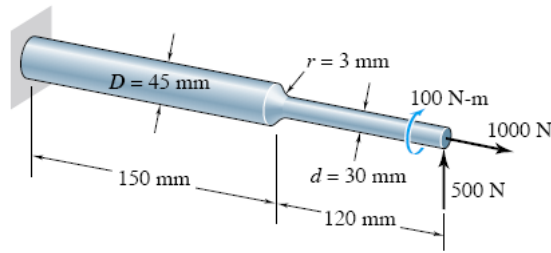


Problem 6.34

The shaft shown in sketch d is subjected to tensile, torsional, and bending loads. Determine the principal stresses at the location of stress concentration.

Ans. $\sigma_1 = 53.2 \text{ MPa}$, $\sigma_2 = 0$, $\sigma_3 = -13.2 \text{ MPa}$.



Sketch d, for Problem 6.34

Notes: This problem can be easily solved through the principle of superposition. The stress concentration factors are obtained from Figures 6.5 (a), (b), and (c).

Solution: The rod will see normal stresses due to axial loads and bending, and a shear stress due to torsion. Note that the shear stress due to transverse (bending) shear is zero at the extreme fibers where the other stresses are largest. The critical location is at the bottom where the bending and axial stresses are both tensile. Assign the x-axis to the rod axis. The normal stress is given by:

$$\sigma_c = K_{c1} \frac{P}{A} + K_{c2} \frac{Mc}{I} = (1.9) \frac{1000 \text{ N}}{\frac{\pi}{4} (0.03 \text{ m})^2} + (1.65) \frac{(500 \text{ N})(0.120 \text{ m})(0.015 \text{ m})}{\frac{\pi}{64} (0.030 \text{ m})^4} = 40.0 \text{ MPa}$$

where the stress concentration factors of 1.9 and 1.65 are obtained from Figure 6.5 (a) and (b).

The shear stress is

$$\tau = K_c \frac{Tc}{J} = (1.4) \frac{(100 \text{ Nm})(0.015 \text{ m})}{\frac{\pi}{32} (0.030 \text{ m})^4} = 26.45 \text{ MPa}$$

where the stress concentration factor of 1.4 is obtained from Figure 6.5 (c).

Equation (2.16) gives the principal stresses.

$$\begin{aligned} \sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2} \right)^2} \\ &= \frac{40.0 \text{ MPa}}{2} \pm \sqrt{(26.45 \text{ MPa})^2 + \left(\frac{40.0 \text{ MPa}}{2} \right)^2} = 20 \pm 33.16 \text{ MPa} \end{aligned}$$

or $\sigma_1 = 53.2 \text{ MPa}$ and $\sigma_2 = -13.2 \text{ MPa}$. Note that the shear stress is very small compared to the normal stress; we could have taken σ_x as a principal direction.

Problem 6.56

A bolt is tightened, subjecting its shank to a tensile stress of 80 ksi and a torsional shear stress of 50 ksi at a critical point. All of the other stresses are zero. Find the safety factor at the critical point by the DET and the MSST. The material is high-carbon steel (AISI 1080, Q&T 800 °C). Will the bolt fail because of the static loading? Ans. $n_{s,DET} = 0.47$, $n_{s,MSST} = 0.43$.

Notes: Equations (2.16), (6.7), and (6.10) are used to solve this problem.

Solution: From the inside front cover, the yield stress for this AISI 1080 steel is 55 ksi.

Directions are arbitrary; let's refer to the tensile stress as $\sigma_x = 80$ ksi and the shear stress as $\tau_{xy} = 50$ ksi. Since all other stresses are zero, Eq. (2.16) gives the principal stresses as

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \frac{80 \text{ ksi}}{2} \pm \sqrt{(50 \text{ ksi})^2 + \left(\frac{80 \text{ ksi}}{2}\right)^2}$$

or $\sigma_1 = 104$ ksi, $\sigma_2 = -24$ ksi. Note that the other stresses are zero, so the principal stress out of the plane of the normal and shear stresses is zero. Putting the stresses in the proper order ($\sigma_1 \geq \sigma_2 \geq \sigma_3$), we assign them the values $\sigma_1 = 104$ ksi, $\sigma_2 = 0$ ksi, $\sigma_3 = -24$ ksi. From Eq. (6.7),

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_s} \rightarrow n_s = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{55 \text{ ksi}}{104 \text{ ksi} - (-24 \text{ ksi})} = 0.43$$

which is the safety factor for the maximum shear stress theory. Equation (6.10) gives

$$\begin{aligned} \sigma_e &= \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2} \\ &= \frac{1}{\sqrt{2}} [(104 \text{ ksi} - 0)^2 + (104 \text{ ksi} - (-24 \text{ ksi}))^2 + (0 - (-24 \text{ ksi}))^2]^{1/2} \\ &= 118 \text{ ksi} \end{aligned}$$

From Eq. (6.10),

$$\sigma_e = \frac{S_y}{n_s} \rightarrow n_s = \frac{S_y}{\sigma_e} = \frac{55 \text{ ksi}}{118 \text{ ksi}} = 0.47$$

Since the safety factor is less than one for both cases, both cases predict failure.

Problem 6.62

Use the MSST and the DET to determine the safety factor for **2024-T351** aluminum alloy for each of the following stress states:

- (a) $\sigma_x = 10 \text{ MPa}$, $\sigma_y = -60 \text{ MPa}$ Ans. $n_{s,\text{MSST}} = 4.64$.
 (b) $\sigma_x = \sigma_y = \tau_{xy} = -30 \text{ MPa}$ Ans. $n_{s,\text{DET}} = 5.42$.
 (c) $\sigma_x = -\sigma_y = 20 \text{ MPa}$, and $\tau_{xy} = 10 \text{ MPa}$ Ans. $n_{s,\text{MSST}} = 7.27$.
 (d) $\sigma_x = 2\sigma_y = -70 \text{ MPa}$, and $\tau_{xy} = 40 \text{ MPa}$ Ans. $n_{s,\text{DET}} = 3.58$.

Solution: From Table 6.1, the yield strength for 2024-T351 is $S_y = 325 \text{ MPa}$.

- (a) For $\sigma_x = 10 \text{ MPa}$, $\sigma_y = -60 \text{ MPa}$, note that there are no shear stresses. Therefore, we can directly write the principal stresses as $\sigma_1 = 10 \text{ MPa}$, $\sigma_2 = 0 \text{ MPa}$ and $\sigma_3 = -60 \text{ MPa}$. Note that the principal stresses have been renumbered so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. From Eq. (6.7), the safety factor for MSST is:

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_s}$$

or

$$n_s = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{325 \text{ MPa}}{10 \text{ MPa} + 60 \text{ MPa}} = 4.64$$

Equation (6.10) gives the effective stress as

$$\begin{aligned} \sigma_e &= \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2} \\ &= \frac{1}{\sqrt{2}} [(10 \text{ MPa} - 0)^2 + (10 \text{ MPa} + 60 \text{ MPa})^2 + (60 \text{ MPa})^2]^{1/2} \\ &= 65.57 \text{ MPa} \end{aligned}$$

From Eq. (6.10), the safety factor for DET is:

$$\sigma_e = \frac{S_y}{n_s}$$

or

$$n_s = \frac{S_y}{\sigma_e} = \frac{325 \text{ MPa}}{65.57 \text{ MPa}} = 4.96$$

- (b) For $\sigma_x = \sigma_y = \tau_{xy} = -30 \text{ MPa}$, Eq. (2.16) gives

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \frac{-30 \text{ MPa} - 30 \text{ MPa}}{2} \pm \sqrt{(30 \text{ MPa})^2 + (0)^2}$$

Therefore, $\sigma_1 = 0 \text{ MPa}$, $\sigma_2 = 0 \text{ MPa}$ and $\sigma_3 = -60 \text{ MPa}$. Note that the principal stresses have been renumbered so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. From Eq. (6.7), the safety factor for MSST is:

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_s} \rightarrow n_s = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{325 \text{ MPa}}{0 \text{ MPa} + 60 \text{ MPa}} = 5.42$$

Equation (6.10) gives the effective stress as

$$\begin{aligned} \sigma_e &= \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2} \\ &= \frac{1}{\sqrt{2}} [(0 \text{ MPa})^2 + (60 \text{ MPa})^2 + (60 \text{ MPa})^2]^{1/2} \\ &= 60 \text{ MPa} \end{aligned}$$

From Equation (6.10), the safety factor for DET is:

$$\sigma_e = \frac{S_y}{n_s} \rightarrow n_s = \frac{S_y}{\sigma_e} = \frac{325 \text{ MPa}}{60 \text{ MPa}} = 5.42$$

(c) For $\sigma_x = -\sigma_y = 36$ MPa and $\tau_{xy} = 65$ MPa, Eq. (2.16) gives

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} \\ &= \frac{36 \text{ MPa} - 36 \text{ MPa}}{2} \pm \sqrt{(65 \text{ MPa})^2 + (36 \text{ MPa} + 36 \text{ MPa})^2}\end{aligned}$$

Therefore, $\sigma_1 = 97$ MPa, $\sigma_2 = 0$ MPa and $\sigma_3 = -97$ MPa. Note that the principal stresses have been renumbered so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. From Eq. (6.7), the safety factor for MSST is:

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_s}$$

or

$$n_s = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{325 \text{ MPa}}{97 \text{ MPa} + 97 \text{ MPa}} = 1.67$$

Equation (6.10) gives the effective stress as

$$\begin{aligned}\sigma_e &= \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2} \\ &= \frac{1}{\sqrt{2}} [(97 \text{ MPa} - 0)^2 + (97 \text{ MPa} + 97 \text{ MPa})^2 + (97 \text{ MPa})^2]^{1/2} \\ &= 168.0 \text{ MPa}\end{aligned}$$

From Eq. (6.10), the safety factor for DET is:

$$\sigma_e = \frac{S_y}{n_s}$$

or

$$n_s = \frac{S_y}{\sigma_e} = \frac{325 \text{ MPa}}{168.0 \text{ MPa}} = 1.93$$

(d) For $\sigma_x = 2\sigma_y = -112$ MPa, and $\tau_{xy} = 33$ MPa, Eq. (2.16) gives

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} \\ &= \frac{-112 \text{ MPa} - 56 \text{ MPa}}{2} \pm \sqrt{(33 \text{ MPa})^2 + (-112 \text{ MPa} + 56 \text{ MPa})^2}\end{aligned}$$

Therefore, $\sigma_1 = 0$ MPa, $\sigma_2 = -19$ MPa and $\sigma_3 = -149$ MPa. Note that the principal stresses have been renumbered so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. From Eq. (6.7), the safety factor for MSST is:

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_s}$$

or

$$n_s = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{325 \text{ MPa}}{0 \text{ MPa} + 149 \text{ MPa}} = 2.181$$

Equation (6.10) gives the effective stress as

$$\begin{aligned}\sigma_e &= \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2} \\ &= \sqrt{1/2} [(19 \text{ MPa})^2 + (149 \text{ MPa})^2 + (-19 \text{ MPa} + 149 \text{ MPa})^2]^{1/2} \\ &= 140.8 \text{ MPa}\end{aligned}$$

From Eq. (6.10), the safety factor for DET is:

$$\sigma_e = \frac{S_y}{n_s}$$

or

$$n_s = \frac{S_y}{\sigma_e} = \frac{325 \text{ MPa}}{140.8 \text{ MPa}} = 2.31$$