# Solutions to Practice Problems for Exam 3

Calculus I, MATH 1141

Fall 2020

Read each question carefully and practice showing all work to earn full credit.

1. Find the absolute maximum and minimum values of the function on the given interval.

a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$

*Solution.* $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1)$.

$$0 = 12x(x-2)(x+1) \implies x = 0, 2, -1 \quad \text{critical numbers}$$

$$\text{critical values: } f(0) = 1,\ f(2) = -31,\ f(-1) = -4$$

$$\text{endpoint values: } f(-2) = 33,\ f(3) = 28$$

So $f$ has an absolute maximum value of 33 at $x = -2$, and $f$ has an absolute minimum value of $-31$ at $x = 2$. □

b) $f(x) = 1/x$ on the interval $[1, \infty)$

*Solution.* $f'(x) = -1/x^2$. $f'(x)$ is never zero so there are no critical numbers. In fact, $f'(x) < 0$ on the interval $[1, \infty)$ so that $f$ is decreasing on this interval. Thus the absolute maximum occurs at the left endpoint: $f(1) = 1$. But there is no absolute minimum since $f(x)$ approaches the horizontal asymptote $y = 0$ as $x \to \infty$. □

c) $f(x) = \dfrac{e^x}{1+x^2}$ on the interval $[0, 3]$

*Solution.*

$$f'(x) = \frac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2} = \frac{e^x(x^2 - 2x + 1)}{(1+x^2)^2} = \frac{e^x(x-1)(x-1)}{(1+x^2)^2}$$

$$0 = f'(x) \implies 0 = e^x(x-1)(x-1) \implies x = 1 \quad \text{critical number}$$

$$\text{critical value: } f(1) = \tfrac{e}{2}$$

$$\text{endpoint values: } f(0) = 1,\ f(3) = \tfrac{e^3}{10}$$

So $f$ has an absolute maximum value of $\frac{e^3}{10}$ at $x = 3$, and $f$ has an absolute minimum value of 1 at $x = 0$. □

d) $f(t) = 1 + \cos^2(t)$ on the interval $[\frac{\pi}{4}, \pi]$.

*Solution.* $f'(t) = -2\cos(t)\sin(t)$

$$0 = -2\cos(t)\sin(t) \implies \text{either } \cos(t) = 0 \text{ or } \sin(t) = 0 \implies \text{either } t = \pi/2 \text{ or } t = \pi$$

$$\text{critical values: } f(\pi/2) = 1 + \cos^2(\pi/2) = 1,\ f(\pi) = 1 + \cos^2(\pi) = 1 + (-1)^2 = 2$$

$$\text{endpoint values: } f(\pi/4) = 1 + \cos^2(\pi/4) = 1 + (\tfrac{\sqrt{2}}{2})^2 = \tfrac{3}{2},\ f(\pi) = 1 + \cos^2(\pi) = 2$$

So $f$ has an absolute maximum value of 2 at $t = \pi$, and $f$ has an absolute minimum value of 1 at $t = \pi/2$. □

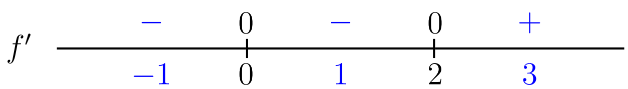
2. For each function, find the intervals on which $f$ is increasing or decreasing. Find all local maximum and minimum values of $f$. Be sure to state both the critical numbers and the critical values.

a) $f(x) = 6x^4 - 16x^3 + 1$

*Solution.* $f'(x) = 24x^3 - 48x^2 = 24x^2(x-2)$

$$0 = 24x^2(x-2) \implies x = 0, 2 \quad \text{critical numbers;} \qquad \text{critical values: } f(0) = 1,\ f(2) = -31$$

First derivative sign chart.




$$f'(-1) = 24(-1)^2(-3) < 0$$
$$f'(1) = 24(1)^2(-1) < 0$$
$$f'(3) = 24(3)^2(1) > 0$$

$f$ is increasing on $(2, \infty)$. $f$ is decreasing on $(-\infty, 2)$.

$f$ has a local minimum of $-31$ at $x = 2$. $f$ has no local maximum. $f$ has neither a local maximum nor a local minimum at $x = 0$. □

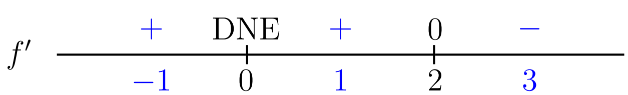
b) $f(x) = 8x^{1/3} + x^{4/3}$

*Solution.* $f'(x) = \frac{8}{3}x^{-2/3} + \frac{4}{3}x^{1/3} = \frac{4}{3}x^{-2/3}(2-x) = \frac{4(2-x)}{3x^{2/3}}$

$$0 = \frac{4(2-x)}{3x^{2/3}} \implies \text{critical numbers: } x = 0\ (f'(0) \text{ DNE}),\ x = 2\ (f'(2) = 0)$$

$$\text{critical values: } f(0) = 0,\ f(2) = 8\sqrt[3]{2} + 2\sqrt[3]{2} = 10\sqrt[3]{2}$$

First derivative sign chart.




$$f'(-1) = \frac{4(3)}{3(-1)^{2/3}} > 0$$
$$f'(1) = \frac{4(1)}{3(1)^{2/3}} > 0$$
$$f'(3) = \frac{4(-1)}{3(3)^{2/3}} < 0$$

$f$ is increasing on $(-\infty, 2)$. $f$ is decreasing on $(2, \infty)$.

$f$ has a local maximum of $10\sqrt[3]{2}$ at $x = 2$. $f$ has no local minimum. $f$ has neither a local maximum nor a local minimum at $x = 0$. □

c) $f(x) = \ln(2 + \sin(x)), \qquad 0 \le x \le 2\pi$

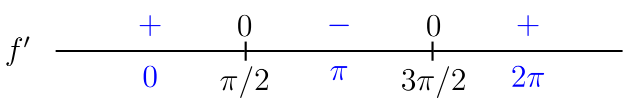
*Solution.* Using the Chain Rule, $f'(x) = \frac{1}{2+\sin(x)} \cdot \cos(x) = \frac{\cos(x)}{2+\sin(x)}$

$$0 = \frac{\cos(x)}{2+\sin(x)} \implies \cos(x) = 0 \implies x = \pi/2, 3\pi/2 \quad \text{critical numbers}$$

$$\text{critical values: } f(\pi/2) = \ln(3),\ f(3\pi/2) = \ln(1) = 0$$

Notice that the denominator of $f'(x)$ is never 0 since $-1 \le \sin(x) \le 1$ so $f'(x)$ is defined everywhere.

First derivative sign chart.




$$f'(0) = \tfrac{\cos(0)}{2+\sin(0)} > 0$$
$$f'(\pi) = \tfrac{\cos(\pi)}{2+\sin(\pi)} < 0$$
$$f'(2\pi) = \tfrac{\cos(2\pi)}{2+\sin(2\pi)} > 0$$

$f$ is increasing on $(0, \pi/2), (3\pi/2, 2\pi)$. $f$ is decreasing on $(\pi/2, 3\pi/2)$.
$f$ has a local maximum of $\ln(3)$ at $x = \pi/2$. $f$ has a local minimum of 0 at $x = 3\pi/2$. $\square$

3. For each function, find the intervals on which $f$ is concave up and concave down. Find all inflection points for $f$.

a) $f(x) = e^{-x^2}$

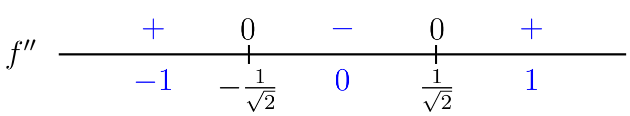
*Solution.* Using the Chain Rule, $f'(x) = e^{-x^2}(-2x) = -2xe^{-x^2}$. Using the Product Rule,

$$f''(x) = -2 \cdot e^{-x^2} - 2x \cdot e^{-x^2}(-2x) = e^{-x^2}(4x^2 - 2)$$

$$0 = e^{-x^2}(4x^2 - 2) \implies x^2 = \tfrac{1}{2} \implies x = \pm\tfrac{1}{\sqrt{2}}$$

$$y \text{ values: } f(\tfrac{1}{\sqrt{2}}) = e^{-1/2},\ f(-\tfrac{1}{\sqrt{2}}) = e^{-1/2}$$

Second derivative sign chart.




$$f''(-1) = e^{-1}(2) > 0$$
$$f''(0) = e^{0}(-2) < 0$$
$$f''(1) = e^{-1}(2) > 0$$

$f$ is concave up on $(-\infty, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, \infty)$. $f$ is concave down on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
Since the concavity changes sign, $f$ has points of inflection at $(-\frac{1}{\sqrt{2}}, f(-\frac{1}{\sqrt{2}})) = (-\frac{1}{\sqrt{2}}, e^{-1/2})$ and $(\frac{1}{\sqrt{2}}, f(\frac{1}{\sqrt{2}})) = (\frac{1}{\sqrt{2}}, e^{-1/2})$. $\square$

b) $f(x) = x^2\ln(x)$, $x > 0$

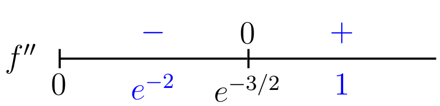
*Solution.* Using the Product Rule, $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x\ln(x) + x$. Using the Product Rule again,

$$f''(x) = 2 \cdot \ln(x) + 2x \cdot \tfrac{1}{x} + 1 = 2\ln(x) + 3$$

$$0 = 2\ln(x) + 3 \implies \ln(x) = -\tfrac{3}{2} \implies x = e^{-3/2}$$

$$y \text{ values: } f(e^{-3/2}) = (e^{-3/2})^2\ln(e^{-3/2}) = -\tfrac{3}{2}e^{-3}$$

Second derivative sign chart.




$$f''(e^{-2}) = 2\ln(e^{-2}) + 3 = -1 < 0$$
$$f''(1) = 2\ln(1) + 3 = 3 > 0$$

$f$ is concave down on $(0, e^{-3/2})$. $f$ is concave up on $(e^{-3/2}, \infty)$.
Since the concavity changes sign, $f$ has a point of inflection at
$(e^{-3/2}, f(e^{-3/2})) = (e^{-3/2}, -\frac{3}{2}e^{-3})$. □

c) $f(x) = \dfrac{1}{x^2+1}$

*Solution.* Using the Quotient Rule, $f'(x) = \frac{(x^2+1)(0)-(1)(2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$. Using the Quotient Rule again,

$$f''(x) = \frac{(x^2+1)^2(-2) - (-2x)(2(x^2+1)(2x))}{(x^2+1)^4} = \frac{(x^2+1)[-2(x^2+1)+8x^2]}{(x^2+1)^4} = \frac{6x^2-2}{(x^2+1)^3}$$

$$0 = \frac{6x^2-2}{(x^2+1)^3} \implies 6x^2 - 2 = 0 \implies x^2 = \tfrac{1}{3} \implies x = \pm\tfrac{1}{\sqrt{3}}$$

$$y \text{ values: } f(\tfrac{1}{\sqrt{3}}) = \frac{1}{\frac{1}{3}+1} = \frac{3}{4},\ f(-\tfrac{1}{\sqrt{3}}) = \frac{1}{\frac{1}{3}+1} = \frac{3}{4}$$

Second derivative sign chart.

| $f''$ | $+$ | $0$ | $-$ | $0$ | $+$ |
|---|---|---|---|---|---|
| $x$ | $-1$ | $-\frac{1}{\sqrt{3}}$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ |

$f''(-1) = \frac{4}{(2)^3} > 0$
$f''(0) = \frac{-2}{(1)^3} < 0$
$f''(1) = \frac{4}{(2)^3} > 0$

$f$ is concave up on $(-\infty, -\frac{1}{\sqrt{3}}), (\frac{1}{\sqrt{3}}, \infty)$. $f$ is concave down on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.
Since the concavity changes sign, $f$ has points of inflection at $(-\frac{1}{\sqrt{3}}, f(-\frac{1}{\sqrt{3}})) = (-\frac{1}{\sqrt{3}}, \frac{3}{4})$ and $(\frac{1}{\sqrt{3}}, f(\frac{1}{\sqrt{3}})) = (\frac{1}{\sqrt{3}}, \frac{3}{4})$. □

4. Evaluate the following limits. Use L'Hospital's Rule where appropriate.

a) $\displaystyle\lim_{x\to\pi/4} \frac{\sin(x)-\cos(x)}{\tan(x)-1}$

*Solution.* $\sin(\pi/4) - \cos(\pi/4) = 0$ and $\tan(\pi/4) - 1 = 0$ so this limit is of the form $0/0$ and we may apply L'Hospital's rule.

$$\lim_{x\to\pi/4} \frac{\sin(x)-\cos(x)}{\tan(x)-1} \overset{L'H}{=} \lim_{x\to\pi/4} \frac{\cos(x)+\sin(x)}{\sec^2(x)} = \frac{\cos(\pi/4)+\sin(\pi/4)}{\sec^2(\pi/4)} = \frac{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}{2} = \frac{\sqrt{2}}{2}$$

□

b) $\displaystyle\lim_{x\to 0} \frac{\tan(5x)}{\sin(3x)}$

*Solution.* $\tan(0) = 0$ and $\sin(0) = 0$ so this limit is of the form $0/0$ and we may apply L'Hospital's rule.

$$\lim_{x\to 0} \frac{\tan(5x)}{\sin(3x)} \overset{L'H}{=} \lim_{x\to 0} \frac{5\sec^2(5x)}{3\cos(3x)} = \frac{5\sec^2(0)}{3\cos(0)} = \frac{5}{3}$$

□

c) $\lim_{t\to 0} \dfrac{e^{2t}-1}{\sin(t)}$

*Solution.* $e^0 - 1 = 0$ and $\sin(0) = 0$ so this limit is of the form $0/0$ and we may apply L'Hospital's rule.

$$\lim_{t\to 0} \frac{e^{2t}-1}{\sin(t)} \overset{L'H}{=} \lim_{t\to 0} \frac{2e^{2t}}{\cos(t)} = \frac{2e^0}{\cos(0)} = \frac{2}{1} = 2$$

□

d) $\lim_{\theta\to 0} \dfrac{1+\cos(\theta)}{1-\cos(\theta)}$

*Solution.* $1+\cos(0) = 2$ and $1-\cos(0) = 0$ so this limit is not of the form $0/0$ or $\infty/\infty$ and we may not apply L'Hospital's rule. In fact,

$$\lim_{\theta\to 0} \frac{1+\cos(\theta)}{1-\cos(\theta)} = +\infty$$

since $1-\cos(\theta) > 0$ for small $\theta \neq 0$. □

e) $\lim_{x\to\infty} (1+\frac{2}{x})^x$

*Solution.* This expression looks like $1^\infty$, which is an indeterminate form that we must manipulate in order to apply L'Hospital's rule.

$$\left(1+\tfrac{2}{x}\right)^x = e^{x\ln(1+\frac{2}{x})} \qquad \text{and} \qquad x\ln(1+\tfrac{2}{x}) = \frac{\ln(1+\frac{2}{x})}{\frac{1}{x}}$$

This last expression is of the form $0/0$ so we may apply L'Hospital's rule.

$$\lim_{x\to\infty} x\ln(1+\tfrac{2}{x}) = \lim_{x\to\infty} \frac{\ln(1+\frac{2}{x})}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{2}{x}} \cdot \frac{-2}{x^2}}{\frac{-1}{x^2}} = \lim_{x\to\infty} \frac{2}{1+\frac{2}{x}} = \frac{2}{1} = 2$$

$$\text{Thus} \quad \lim_{x\to\infty} \left(1+\tfrac{2}{x}\right)^x = \lim_{x\to\infty} e^{x\ln(1+\frac{2}{x})} = e^2$$

since $e^x$ is a continuous function. □

5. Find the general antiderivative of the following functions.

a) $f(x) = x^3 - 5x^2 - 3x + 19$     $F(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 - \frac{3}{2}x^2 + 19x + C$

b) $g(t) = \dfrac{3}{t^2} - 8\sqrt{t} = 3t^{-2} - 8t^{1/2}$     $G(t) = -3t^{-1} - 8\cdot\frac{2}{3}t^{3/2} + C = -\frac{3}{t} - \frac{16}{3}t^{3/2} + C$

c) $f(x) = 3e^x + \dfrac{2}{x}$     $F(x) = 3e^x + 2\ln(x) + C$

d) $f(t) = \sin(t) - 5\sec^2(t)$     $F(t) = -\cos(t) - 5\tan(t) + C$

e) $g(x) = 4\sinh(t) - 3\cosh(x)$     $G(x) = 4\cosh(t) - 3\sinh(t) + C$

f) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$     $F(x) = \sin^{-1}(x) + C$

6. Suppose $f''(x) = -2 + 12x - 12x^2$, $f(0) = 4$ and $f(1) = 6$. Find $f(x)$.

*Solution.*

$$f''(x) = -12x^2 + 12x - 2 \implies f'(x) = -4x^3 + 6x^2 - 2x + C$$
$$\implies f(x) = -x^4 + 2x^3 - x^2 + Cx + D$$

Since $f(0) = 4$, we must have,

$$4 = f(0) = -(0)^4 + 2(0)^3 - (0)^2 + C(0) + D \implies 4 = D$$

So $f(x) = -x^4 + 2x^3 - x^2 + Cx + 4$. Then we use $f(1) = 6$ to get,

$$6 = f(1) = -(1)^4 + 2(1)^3 - (1)^2 + C(1) + 4 \implies 6 = C + 4 \implies C = 2$$

So $f(x) = -x^4 + 2x^3 - x^2 + 2x + 4$. □

7. Suppose the acceleration of an object is given by $a(t) = 2t + 1$. Find the position $x(t)$ if $x(0) = 3$ and $v(0) = -2$.

*Solution.*

$$a(t) = 2t + 1 \implies v(t) = t^2 + t + C$$

Since $v(0) = -2$, we must have,

$$-2 = v(0) = (0)^2 + 0 + C \implies -2 = C \implies v(t) = t^2 + t - 2$$

$$v(t) = t^2 + t - 2 \implies x(t) = \tfrac{1}{3}t^3 + \tfrac{1}{2}t^2 - 2t + C$$

Since $x(0) = 3$, we must have,

$$3 = x(0) = \tfrac{1}{3}(0)^3 + \tfrac{1}{2}(0)^2 - 2(0) + C \implies 3 = C$$
$$\implies x(t) = \tfrac{1}{3}t^3 + \tfrac{1}{2}t^2 - 2t + 3$$

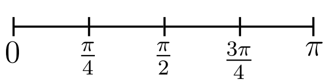
□

8. Approximate the area under the curve $f(x) = \sin^2(x)$ from $x = 0$ to $x = \pi$ using 4 rectangles with function values evaluated at:

a) left endpoints; b) right endpoints; c) midpoints

Round your answers to 4 decimal places. Use your calculator to find the error in each approximation.

*Solution.*




$$\Delta x = \tfrac{\pi - 0}{4} = \tfrac{\pi}{4}$$

a) Left endpoints

$$L_4 = \Delta x\big[f(0) + f(\pi/4) + f(\pi/2) + f(3\pi/4)\big] = \tfrac{\pi}{4}\big[\sin^2(0) + \sin^2(\tfrac{\pi}{4}) + \sin^2(\tfrac{\pi}{2}) + \sin^2(\tfrac{3\pi}{4})\big]$$
$$= \tfrac{\pi}{4}\big[2\big] = \tfrac{\pi}{2} \approx 1.5708$$

In this case, the answer is exact and the error is 0.

b) Right endpoints

$$R_4 = \Delta x\big[f(\pi/4) + f(\pi/2) + f(3\pi/4) + f(\pi)\big] = \tfrac{\pi}{4}\big[\sin^2(\tfrac{\pi}{4}) + \sin^2(\tfrac{\pi}{2}) + \sin^2(\tfrac{3\pi}{4}) + \sin^2(\pi)\big]$$
$$= \tfrac{\pi}{4}\big[2\big] = \tfrac{\pi}{2} \approx 1.5708$$

In this case, the answer is exact and the error is 0.

c) Midpoints

$$M_4 = \Delta x\big[f(\pi/8) + f(3\pi/8) + f(5\pi/8) + f(7\pi/8)\big] = \tfrac{\pi}{4}\big[\sin^2(\tfrac{\pi}{8}) + \sin^2(\tfrac{3\pi}{8}) + \sin^2(\tfrac{5\pi}{8}) + \sin^2(\tfrac{7\pi}{8})\big]$$
$$\tfrac{\pi}{4}\big[2\big] = \tfrac{\pi}{2} \approx 1.5708$$

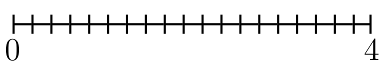
In this case, the answer is exact and the error is 0.

□

9. Express the following integral as the limit of Riemann sums and then evaluate the limit.

$$\int_0^4 (x - x^2)\, dx.$$

*Solution.* $f(x) = x - x^2$




$$\Delta x = \tfrac{4-0}{n} = \tfrac{4}{n}$$
$$x_0 = 0,\ x_n = 4$$
$$x_i = x_0 + i\Delta x = \tfrac{4i}{n}$$

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}(x_i - x_i^2)\cdot \tfrac{4}{n} = \sum_{i=1}^{n}\left(\tfrac{4i}{n} - (\tfrac{4i}{n})^2\right)\cdot\tfrac{4}{n} = \sum_{i=1}^{n}\tfrac{16i}{n^2} - \tfrac{64i^2}{n^3}$$

Now $$\sum_{i=1}^{n}\frac{16i}{n^2} = \frac{16}{n^2}\sum_{i=1}^{n} i = \frac{16}{n^2}\cdot\frac{n(n+1)}{2} = \frac{16n^2+16n}{2n^2} = 8 + \frac{8}{n}$$

and $$\sum_{i=1}^{n}\frac{64i^2}{n^3} = \frac{64}{n^3}\sum_{i=1}^{n} i^2 = \frac{64}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{128n^3+192n^2+64n}{6n^3} = \frac{64}{3}+\frac{32}{n}+\frac{32}{3n^2}$$

Putting these into the expression for $R_n$ gives,

$$\int_0^4 (x-x^2)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(8+\frac{8}{n}\right)-\left(\frac{64}{3}+\frac{32}{n}+\frac{32}{3n^2}\right) = 8+0-\left(\frac{64}{3}+0+0\right) = -\frac{40}{3}$$

□

10. Use part one of the Fundamental Theorem of Calculus to find the derivative of each function.

a) $f(x) = \int_x^{2x} e^{t^2}\,dt = \int_x^0 e^{t^2}\,dt + \int_0^{2x} e^{t^2}\,dt = -\int_0^x e^{t^2}\,dt + \int_0^{2x} e^{t^2}\,dt$

By the Chain Rule, $f'(x) = -e^{x^2} + e^{(2x)^2}\cdot 2 = -e^{x^2} + 2e^{4x^2}$

b) $f(x) = \int_1^{\sqrt{x}} \ln(1+t^2)\,dt$

Since $\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$, we again use the Chain Rule:

$$f'(x) = \ln(1 + (\sqrt{x})^2) \cdot \frac{1}{2\sqrt{x}} = \frac{\ln(1+x)}{2\sqrt{x}}$$

11. Use part two of the Fundamental Theorem of Calculus to evaluate the definite integrals.

a) $\displaystyle\int_1^8 x^{-2/3}\,dx = 3x^{1/3}\Big|_1^8 = 3(8)^{1/3} - 3(1)^{1/3} = 6 - 3 = 3$

b) $\displaystyle\int_1^3 \left(\frac{1}{t^2} + \frac{1}{t^3}\right) dt = \int_1^3 (t^{-2} + t^{-3})\,dt = (-t^{-1} - \tfrac{1}{2}t^{-2})\big|_1^3 = \left(-\frac{1}{t} - \frac{1}{2t^2}\right)\Big|_1^3$

$\displaystyle = \left(-\frac{1}{3} - \frac{1}{18}\right) - \left(-1 - \frac{1}{2}\right) = \left(-\frac{7}{18}\right) - \left(-\frac{3}{2}\right) = \frac{20}{18} = \frac{10}{9}$

c) $\displaystyle\int_0^{\pi/2} \sin(\theta)\,d\theta = -\cos(\theta)\Big|_0^{\pi/2} = -\cos(\pi/2) + \cos(0) = -0 + 1 = 1$

d) $\displaystyle\int_0^{\pi/3} \sec(\theta)\tan(\theta)\,d\theta = \sec(\theta)\Big|_0^{\pi/3} = \sec(\pi/3) - \sec(0) = 2 - 1 = 1$

e) $\displaystyle\int_{-3}^3 e\,dx = ex\Big|_{-3}^3 = e3 - e(-3) = 6e$

f) $\displaystyle\int_0^1 \left(e^x - \frac{4}{1+x^2}\right) dx = \left(e^x - 4\tan^{-1}(x)\right)\Big|_0^1 = \left(e^1 - 4\tan^{-1}(1)\right) - \left(e^0 - 4\tan^{-1}(0)\right)$

$= \left(e - 4 \cdot \frac{\pi}{4}\right) - (1 - 4 \cdot 0) = (e - \pi) - (1) = e - \pi - 1$