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Statistics 3352

Problem Set VI

Due April 20.

All write-ups should be individually done, complete, neat and precise. Please tell me anyone you worked with or got help from and AI tools, websites or books you used and exactly how.

1. [4pt] Test at the .05 significance level whether the mean of normal population with a known standard deviation $\sigma = 3.2$ is less than 10, given a sample of $n = 16$ with $\overline{x} = 8.4$. Give the null and alternate hypotheses and the p-value. For each of the assumptions of the test, explain why it is probably met, why it is probably not met, or what additional information you would need to have to see that it was met (for example, "I would need to know that the population had less than 100 individuals," or "I would need to know that the population distribution was bimodal".

2. [4pt] Test at the .02 significance level whether the mean of highly skewed population is more than 50, given a simple random sample of $n = 20$ with $\overline{x} = 49.1$ and $s = 1.3$. Give the null and alternate hypotheses and the p-value. For each of the assumptions of the test, explain why it is probably met, why it is probably not met, or what additional information you would need to have to see that it was met (for example, "I would need to know that the population had less than 100 individuals," or "I would need to know that the population distribution was bimodal".

3. [4pt] You take a simple random sample of 700 likely voters and ask them whether they will vote for your candidate. 380 of them say yes. Give a 95% confidence interval for the proportion of all likely voters who intend to vote for your candidate. Identify the population, variable, parameter, and statistic under consideration, and the value for the statistic. Test the claim at the 5% significance level that your candidate would win the election if held today. Give the null and alternate hypotheses, the p-value, and your conclusion in a simple English sentence. For each of the assumptions of the test, explain why it is probably met, why it is probably not met, or what additional information you would need to have to see that it was met.

4. [3pt] A single observation of a random variable having an exponential distribution is used to test the simple null hypothesis that the mean is $\theta = 2$ against the simple alternate hypothesis that the mean is $\theta = 5$. Suppose our critical region is the observation is ≥ 3 (so we reject the null if $X \geq 3$ and accept it if $X < 3$ find the probabilities of making a Type I error and of making a Type II error.

5. [5pt] Look at example 12.4 in the book, and work to understand it (you will have to look back at Example 12.2 and Example 6.67, if you don't, do not be surprised if you don't know what you are doing!). Repeat it changing the assumption from $\mu_1 > \mu_0$ to $\mu_1 < \mu_0$, and conclude that the Neyman-Pearson Lemma yileds the critical region

$$\overline{X} \leq \mu_0 - z_\alpha \frac{1}{\sqrt{n}}.$$